

MEASURE OF CENTRAL TENDENCY

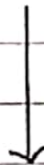
[AVERAGE]

- Measure of central tendency are also usually called as the



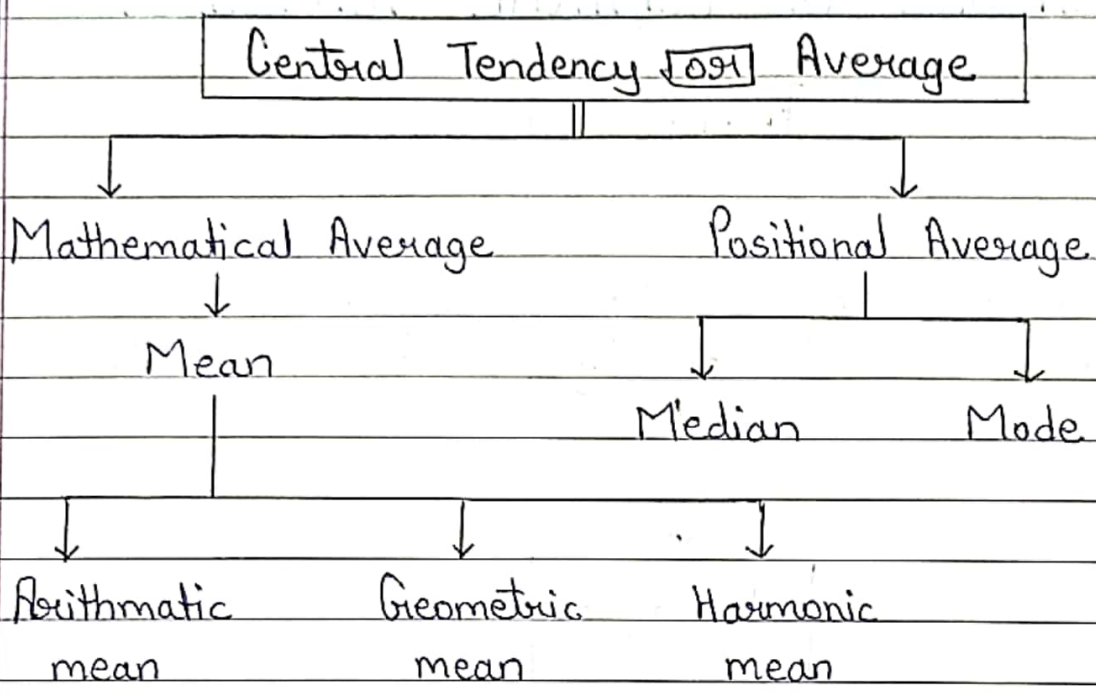
Averages.

- By calculating the measure of central tendency, we can find a single value to represent whole data

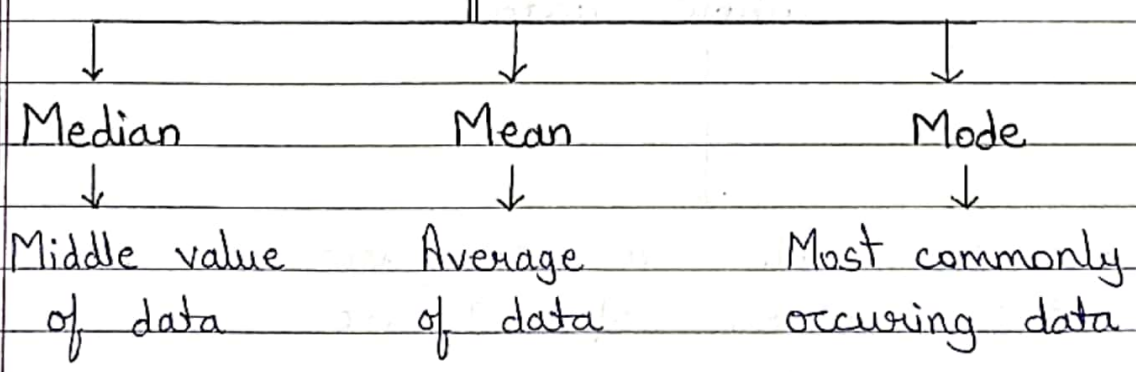


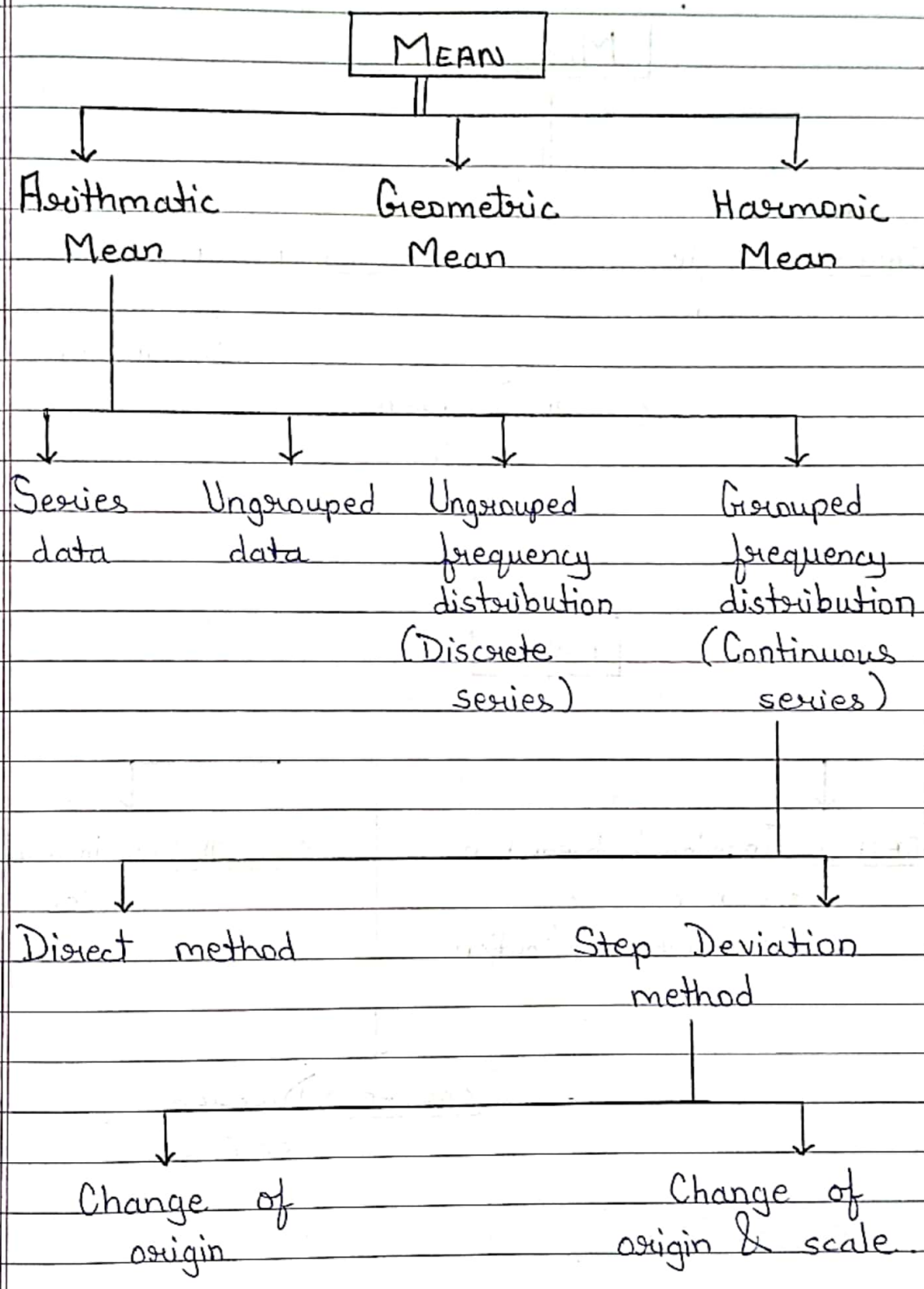
it also help to compare the values of 2 or more groups.

* TYPES OF MEASURE OF CENTRAL TENDENCY :-

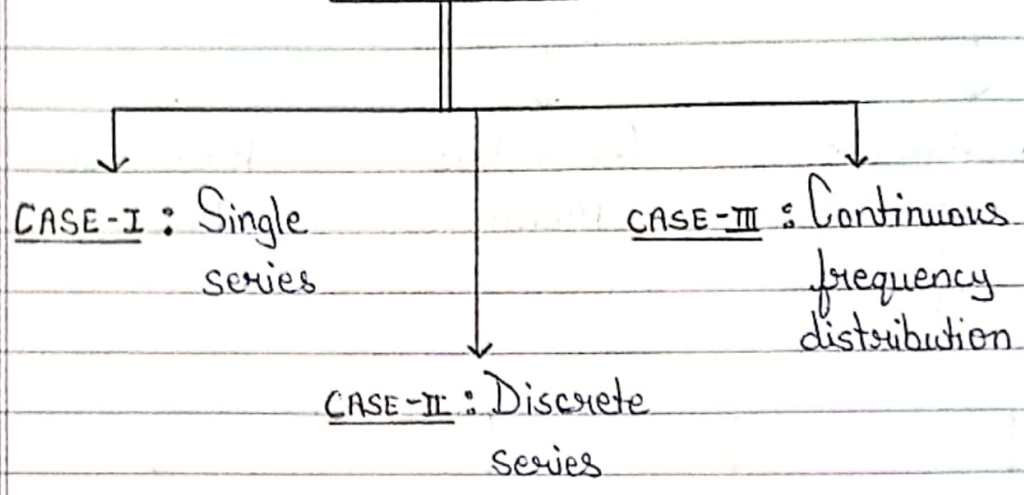


Measure of Central Tendency

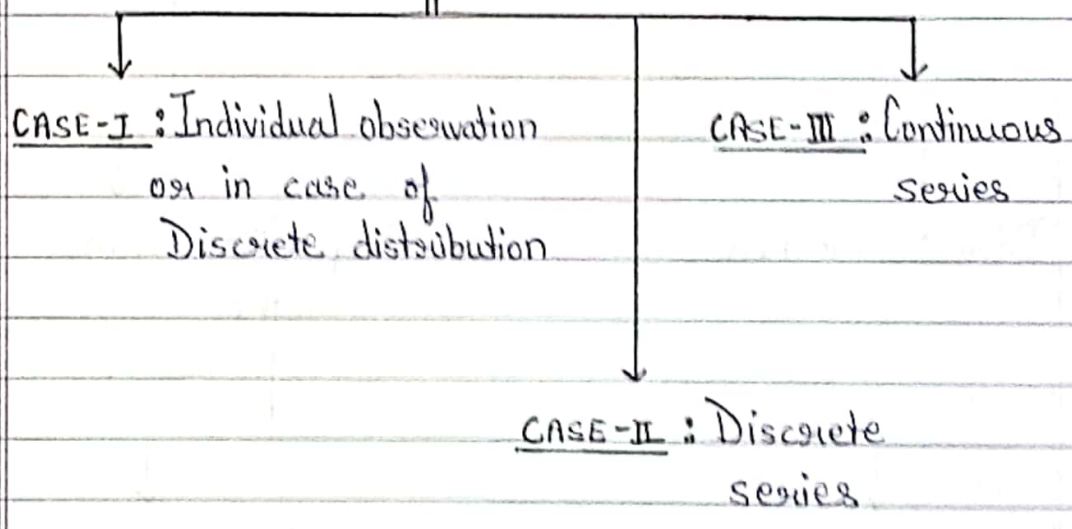




MEDIAN



MODE



FORMULAS FOR MEAN

[I] ARITHMETIC MEAN :-

1) Ungrouped frequency / Series :

$$\bar{x} = \frac{\sum X_i}{n}$$

where,

n = Total no. of observation

X_i = Sum of total no. of observation.

2) Ungrouped frequency distribution

[OR]

[Discrete Series] :

$$\bar{x} = \frac{\sum f_i X_i}{\sum f_i}$$

3) Continuous series [Grouped frequency distribution] :

(i) Direct method :

$$\bar{x} = \frac{\sum fx}{\sum f}$$

(ii) Step-derivation method :

(a) Change of origin :

◦ For ungrouped data :

$$\bar{y} = \frac{y_1 + y_2 + y_3 + \dots + y_n}{n}$$

◦ For ungrouped frequency distribution :

$\bar{y} = \frac{1}{n} \sum fy$	Required A.M = $\bar{x} = A + \bar{y}$
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→ Short-cut method :

$$x = A + \frac{\sum fdx}{\sum f}$$

◦ For grouped frequency distribution :

$\bar{y} = \frac{1}{n} \sum fiyi$	Required A.M = $\bar{x} = A + \bar{y}$
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(b) Change of origin & scale both :

$x_i y_i = \frac{x_i - A}{d}$	$\bar{y} = \frac{1}{n} \sum fiyi$
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Required A.M = $\bar{x} = A + d\bar{y}$

[II] GEOMETRIC MEAN :-

$$G.M = \sqrt{x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n}$$

[III] HARMONIC MEAN :-

$$\frac{1}{H} = \frac{1}{n} \left[\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right]$$

$$= \frac{1}{n} \sum \text{Reciprocal of } x_i$$

$$H = \frac{n}{\sum \text{Reciprocal of } x_i}$$

* EXAMPLES OF SUMS :-

[I] ARITHMETIC MEAN :-

1) For ungrouped data :-

Q: Find the A.M of 3, 6, 5, 8, 6, 11, 13, 19, 12 & 7.

Sol: $A.M = \bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$

$$= \frac{3 + 6 + 5 + 8 + 6 + 11 + 13 + 19 + 12 + 7}{10}$$

$$= \frac{90}{10} = \boxed{9}$$

Q: The birth weight of 6 babies are 2, 2.4, 2.6, 3.1, 3.4 & 2.5 kg. Find the mean birth weight.

Sol: $A.M = \bar{x} = \frac{2 + 2.4 + 2.6 + 3.1 + 3.4 + 2.5}{6}$

$$= \frac{16}{6}$$

$$= \boxed{2.66 \text{ kg}}$$

Q: The labelled potency of a tablet dosage form is 100 mg. The individual tablets are assayed according to a quality control specifications. The results are 101.6, 102, 99.6, 104.7, 103.8, 104.6, 100.5, 106.3, 100.7, 105.2. Find A.M.

Sol: $A.M = \bar{X} = \frac{\sum x_i}{n}$

$$= \frac{101.6 + 102 + 99.6 + 104.7 + 103.8 + 104.6 + 100.5 + 106.3 + 100.7 + 105.2}{10}$$

$$= \frac{1029}{10} = \boxed{102.9}$$

Q: The mean weight of patient in a group of 6 patients is 119. The individual weights of 5 of them are 115, 109, 129, 117 & 114. What is the weight of 6th patient?

Sol: $A.M = \bar{X} = \frac{\sum x_i}{n}$

$$\therefore 119 = \frac{115 + 109 + 129 + 117 + 114 + x}{6}$$

$$\therefore 119 = \frac{584 + x}{6}$$

$$\therefore 119 \times 6 = 584 + x$$

$$\therefore 714 - 584 = x$$

$$\therefore \boxed{x = 130} \rightarrow \text{Weight of 5th patient}$$

2) Ungrouped frequency distribution

[OR]

Discrete Series :

Q: Calculate avg. wage paid per worker from the data given below.

Wages (Rs.) (x)	500	600	700	800	900	1000	1100
Workers (f)	1	3	5	7	6	2	1

Solⁿ:

Wages (x)	f	fx
500	1	500
600	3	1800
700	5	3500
800	7	5600
900	6	5400
1000	2	2000
1100	1	1100
	$\Sigma f = 25$	$\Sigma fx = 19900$

$$\bar{X} = \frac{\sum fx}{\sum f}$$

$$= \frac{19900}{25}$$

∴ $\bar{X} = 796 \text{ Rs}$ → Avg. wage per worker is Rs 796.

Q: The following frequency distribution shows the number of days of confinement of patients after delivery. Find the mean days of confinement.

Day of confinement (x)	6	7	8	9	10
No. of patients (f)	7	6	6	4	2

Solⁿ:

x	f	fx
6	7	42
7	6	42
8	6	48
9	4	36
10	2	20
	$\sum f = 25$	$\sum fx = 188$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{188}{25}$$

$$\therefore \bar{x} = 7.52$$

3] Continuous series [Grouped frequency distribution] :

Q: Find the A.M by direct method of the following data.

Size (x)	3-5	5-7	7-9	9-11	11-13
Frequency (f)	14	16	25	22	12

Sol:

Size of class	Mid-point (xc)	Frequency (f)	fx
3-5	4	14	56
5-7	6	16	96
7-9	8	25	200
9-11	10	22	220
11-13	12	12	144
		$\sum f = 89$	$\sum fx = 716$

$$\bar{x} = \frac{\sum fxc}{\sum f}$$

$$\therefore \bar{x} = \frac{716}{89}$$

$$\therefore \bar{x} = 8.04$$

Q: Compute the avg. marks of students using the following data.

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	3	19	32	17	4

Solⁿ:

Marks	Mid point (x)	Frequency (f)	fx
0-10	5	3	15
10-20	15	19	285
20-30	25	32	800
30-40	35	17	595
40-50	45	4	180
		$\Sigma f = 75$	$\Sigma fx = 1875$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{1875}{75}$$

$$\therefore \bar{x} = 25 \text{ marks}$$

Q: Find the mean weight of 470 infants born in a hospital in one year, from the following table:

Wt. of infants (in Kg)	2-2.4	2.5-2.9	3-3.4	3.5-3.9	4-4.4	4.5-4.9
No. of infants	17	97	187	135	28	6

Solⁿ

Wt. of infants (in Kg)	x	f	fx
2.0-2.4	2.2	17	37.4
2.5-2.9	2.7	97	261.9
3.0-3.4	3.2	187	598.4
3.5-3.9	3.7	135	499.5
4.0-4.4	4.2	28	117.6
4.5-4.9	4.7	6	28.2
		$\Sigma f = 470$	$\Sigma fx = 1543$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1543}{470}$$

$$= 3.28$$

$$\therefore \bar{x} = 3.28 \text{ Kg weight}$$

Q: $x = 3, 6, 5, 8, 6, 11, 13, 19, 12$ & 7 .
Calculate A.M by change of origin method.

Solⁿ: $y_i = x - 3$

$y = 0, 3, 2, 5, 3, 8, 10, 16, 9, 4$.

$\therefore \bar{y} = \frac{0 + 3 + 2 + 5 + 3 + 8 + 10 + 16 + 9 + 4}{10}$

$= \frac{60}{10}$

$\therefore \bar{y} = 6$

Required mean $\bar{x} = 3 + \bar{y} = 3 + 6 = 9$

Q: $\bar{x} = 2, 4, 6, 8, 10, 14, 24, 34$ & 40 .
Calculate A.M by change of origin method.

Solⁿ: $y_i = x - 2$

$y = 0, 2, 4, 8, 12, 32, 22, 38$

$\bar{y} = \frac{0 + 2 + 4 + 8 + 12 + 32 + 22 + 38}{8}$

$\therefore \bar{y} = \frac{118}{8}$

$\therefore \bar{y} = 14.75$

$$\begin{aligned}\text{Required mean} &= \bar{y} + 2 \\ &= 14.75 + 2 \\ &= \boxed{16.75}\end{aligned}$$

Q. Calculate A.M by change of origin method.

Wages (Rs.) (x)	500	600	700	800	900	1000	1100
No. of workers (f)	1	3	5	7	6	2	1

Solⁿ:

Wages (Rs.) x	No. of workers (f)	deviation y = x - 800	fy
500	1	-300	-300
600	3	-200	-600
700	5	-100	-500
<u>800</u> - A	7	0	0
900	6	100	600
1000	2	200	400
1100	1	300	300
	$\Sigma f = 25$		$\Sigma fy = (-100)$

$$\bar{y} = \frac{\Sigma fy}{n} = \frac{-100}{25}$$

$$\therefore \bar{y} = (-4)$$

$$\begin{aligned} \text{Required mean } \bar{x} &= A + \bar{y} \\ &= 800 + (-4) \\ &= 800 - 4 \\ &= \boxed{796} \end{aligned}$$

Q: Calculated A.M by change of origin method.

Class interval	3-5	5-7	7-9	9-11	11-13
Frequency	14	16	25	22	12

Sol ⁿ	Class interval	Mid point (x _i)	f	y _i = x _i - 8	f _i y _i
	3-5	4	14	-4	-56
	5-7	6	16	-2	-32
	7-9	8	25	0	0
	9-11	10	22	2	44
	11-13	12	12	4	48
			n = 89		4

$$\bar{y} = \frac{\sum f y_i}{n} = \frac{4}{89}$$

$$\therefore \bar{y} = \boxed{0.0449}$$

$$\begin{aligned} \text{Required mean } \bar{x} &= 8 + \bar{y} \\ &= 8 + 0.0449 \\ &= \boxed{8.045} \end{aligned}$$

Q: Calculate the mean by change of origin or short-cut method.

Class (x)	10	20	30	40	50	60
Frequency (f)	3	2	5	10	11	8

Solⁿ:

Class (x)	Frequency (f)	dx = x - A	fdx
10	3	-30	-90
20	2	-20	-40
30	5	-10	-50
40 = A	10	0	0
50	11	10	110
60	8	20	160
	$\Sigma f = 39$		$\Sigma fdx = 90$

$$\bar{x} = A + \frac{\Sigma fdx}{\Sigma f}$$

$$= 40 + \frac{90}{39}$$

$$= 40 + 2.31$$

$$\therefore \bar{x} = 42.31$$

Q: The following table shows a distribution of children per family of 191 families. Find the average number of children per family by short-cut method.

No. of children	0	1	2	3	4	5	6	7	8
No. of families	25	16	36	48	30	16	15	3	2

<u>Sol:</u>	No. of children (x)	f	dx = x - A	fdx
	0	25	-4	-100
	1	16	-3	-48
	2	36	-2	-72
	3	48	-1	-48
	4 = A	30	0	0
	5	16	1	16
	6	15	2	30
	7	3	3	9
	8	2	4	8
		$\Sigma f = 191$		$\Sigma fdx = (-205)$

$$\bar{x} = A + \frac{\Sigma fdx}{\Sigma f} = 4 + \frac{(-205)}{191}$$

$$= 4 + (-1.07)$$

$$= 4 - 1.07$$

$$\therefore \bar{x} = 2.93$$

Q: The following table shows a frequency distribution of ages of patients examined on a particular day in the OPD of a hospital. Find the average age of the patients by short-cut method.

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of patients	23	57	138	277	105	74	21

Sol:

Age in years	Mid-point (x)	f_i	$y_i = x_i - 35$	$f_i y_i$
0-10	5	23	-30	-690
10-20	15	57	-20	-1140
20-30	25	138	-10	-1380
30-40	<u>35</u> = A	277	0	0
40-50	45	105	10	1050
50-60	55	74	20	1480
60-70	65	21	30	630
		$\Sigma f = 695$		$\Sigma f_i y_i = (-50)$

$$\bar{y} = \frac{\Sigma f_i y_i}{\Sigma f} = \frac{-50}{695} = (-0.0719)$$

$$\text{Required mean } \bar{x} = 35 + \bar{y}$$

$$= 35 - 0.0719$$

$$\bar{x} = 34.92$$

Q: Given below is the distribution of 140 candidates obtaining marks x or higher in a certain examination.

x	10	20	30	40	50	60	70	80	90	100
C.f.	140	133	118	100	75	45	25	9	2	0

Calculate the mean of marks obtained by short-cut method.

<u>Sol.</u>	Class	Midpoint (x_i)	f_i	$d = \frac{x_i - A}{c}$	$f_i d$
	10-20	15	$140 - 133 = 7$	-4	-28
	20-30	25	$133 - 118 = 15$	-3	-45
	30-40	35	$118 - 100 = 18$	-2	-36
	40-50	45	$100 - 75 = 25$	-1	-25
	50-60	$\boxed{55} = A$	$75 - 45 = 30$	0	0
	60-70	65	$45 - 25 = 20$	1	20
	70-80	75	$25 - 9 = 16$	2	32
	80-90	85	$9 - 2 = 7$	3	21
	90-100	95	$2 - 0 = 2$	4	8
			$\Sigma f = 140$		$\Sigma f_i d = (-53)$

$$\bar{x} = A + \frac{\Sigma f_i d}{\Sigma f} \times c$$

$$= 55 + \frac{(-53)}{140} \times 10$$

$$= 55 + (-0.33) \times 10$$

$$= 55 + (-3.7)$$

$$= 55 - 3.77$$

$$\bar{x} = 51.23$$

Q: From the following distribution of marks obtained by 50 students in quantitative methods. Calculate A.M.

Marks	>10	20	30	40	50	60
No. of students	50	46	40	20	10	3

<u>Solⁿ</u>	Class interval	Mid point	f_i	$y_i = \frac{x-45}{10}$	$f_i y_i$
	10-20	15	4	-3	-12
	20-30	25	6	-2	-12
	30-40	35	20	-1	-20
	40-50	45 = A	10	0	0
	50-60	55	7	1	7
	60-70	65	3	2	6
			$\Sigma f = 50$		$\Sigma f_i y_i = (-31)$

$$\bar{y} = \frac{1}{n} \Sigma f_i y_i = \frac{1}{50} \times (-31) = (-0.62)$$

Required A.M = $\bar{x} = A + d\bar{y}$
 $= 45 + 10(-0.62)$

$\bar{x} = 38.8$ marks

8: Following table shows/gives the cumulative frequency distribution of the ages of 200 teachers. Find the mean of distribution.

Age in years	Cumulative frequency
20 - 25	22
25 - 30	41
30 - 35	91
35 - 40	132
40 - 45	147
45 - 50	167
50 - 55	177
55 - 60	187
60 - 65	199
65 - 70	200

Soln

Class	Mid-point (x_i)	f_i	$d = \frac{x_i - A}{c}$	$f_i d$
20-25	22.5	22	-4	-88
25-30	27.5	19	-3	-57
30-35	32.5	50	-2	-100
35-40	37.5	41	-1	-41
40-45	42.5 = A	15	0	0
45-50	47.5	20	1	20
50-55	52.5	10	2	20
55-60	57.5	10	3	30
60-65	62.5	12	4	48
65-70	67.5	1	5	5
		$\Sigma f = 200$		$\Sigma f_i d = (-163)$

$$\bar{x} = A + \frac{\Sigma f_i d}{\Sigma f} \times c$$

$$= 42.5 + \frac{(-163)}{200} \times 5$$

$$= 42.5 + (-0.815) \times 5$$

$$= 42.5 - 4.075$$

$$\therefore \bar{x} = 38.425$$

Q. Calculate the A.M by change of origin & scale.

Class interval	3-5	5-7	7-9	9-11	11-13
f	14	16	25	22	12

Sol.

Class interval	Mid point (x)	f_i	$d = \frac{x_i - A}{c}$	$f_i d$
3-5	4	14	-2	-28
5-7	6	16	-1	-16
7-9	$\boxed{8} = A$	25	0	0
9-11	10	22	1	22
11-13	12	12	2	24
		$\Sigma f_i = 89$		$\Sigma f_i d = 2$

$$\bar{x} = A + \frac{\Sigma f_i d}{\Sigma f} \times c$$

$$= 8 + \frac{2}{89} \times 2$$

$$= 8 + 0.022 \times 2$$

$$= 8 + 0.044$$

$$\therefore \boxed{\bar{x} = 8.044}$$

[II] GEOMETRIC MEAN :-

1) 15, 5415

$$G.M = \sqrt{x_1 \cdot x_2}$$

$$= \sqrt{15 \times 5415}$$

$$= \sqrt{81255}$$

$$G.M = 285$$

2) 5, 6, 25, 5/6

$$G.M = \sqrt[4]{x_1 \cdot x_2 \cdot x_3 \cdot x_4}$$

$$= \sqrt[4]{5 \times 6 \times 25 \times \frac{5}{6}}$$

$$= \sqrt[4]{5 \times 25 \times 5}$$

$$= \sqrt[4]{5^4}$$

$$\therefore G.M = 5$$

[III] HARMONIC MEAN :-

Q: Find the harmonic mean of $5\frac{1}{2}$, 5, 10 & $10\frac{1}{3}$.

Solⁿ:
$$H.M = \frac{n}{\sum \frac{1}{x_i}}$$

$$= \frac{4}{\frac{2}{5} + \frac{1}{5} + \frac{1}{10} + \frac{3}{10}}$$

$$= \frac{4}{\frac{2}{5} + \frac{1}{5} + \frac{1}{10} + \frac{3}{10}}$$

$$= \frac{4}{\frac{4}{10} + \frac{2}{10} + \frac{1}{10} + \frac{3}{10}}$$

$$= \frac{4}{\frac{10}{10}}$$

$$= \frac{4}{1}$$

\therefore $H.M = 4$

MEDIAN

* CALCULATION OF MEDIAN :-

[I] CASE : SINGLE SERIES

→ Calculation of median involves following steps:

(a) Rearrange data in ascending/decending order of magnitude.

(b) If no. of observation in series is odd,

$$\text{Median value} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ item}$$

(c) If no. of observation in series is even,

$$\text{Median value} = \text{avg. of } \left(\frac{n}{2}\right)^{\text{th}} \text{ \& } \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ item}$$

Q. Find the mean value of data given below:

Income (Rs.) : 1100, 1150, 1080, 1120, 1200, 1160, 1400.

Sol. Rearrange the data in ascending order

↓

1080, 1100, 1120, 1150, 1160, 1200, 1400

Here, $n = 7 = \text{odd}$

$$\text{Median value} = \frac{n+1}{2}$$

$$= \frac{7+1}{2} = \frac{8}{2}$$

= 4th in order

$$\therefore \text{Median value} = 1150 \text{ Rs}$$

Q: Find out the median value of the following data:

10, 9, 11, 12, 6, 8, 13 & 18.

Sol: The arranged series is:

6, 8, 9, 10, 11, 12, 13 & 18.

Here, $n = 8$ i.e. even number.

Median value = avg. of $\left(\frac{n}{2}\right)^{\text{th}}$ & $\left(\frac{n+1}{2}\right)^{\text{th}}$ item

= avg. of 4th & 5th item

$$= \frac{10+11}{2} = \frac{21}{2}$$

$$= 10.5$$

Q: The following figures show incubation periods of 9 polio cases. Find the median of data.

23, 19, 21, 17, 22, 18, 20, 24 & 16.

Solⁿ Rearrange the data in ascending order.

↓

16, 17, 18, 19, 20, 21, 22, 23, 24

Here, $n = 9 = \text{odd}$.

$$\text{Median value} = \left(\frac{n+1}{2} \right)$$

$$= \frac{9+1}{2}$$

$$= \frac{10}{2}$$

$$= 5^{\text{th}} \text{ item}$$

$$\therefore \boxed{\text{Median value} = 20}$$

Q: Find the median of data set :

102, 403, 729, 843, 920, 360, 842, 941, 357,
483, 207, 670, 431, 101.

Solⁿ. Arrange data in ascending order.



102, 109, 207, 357, 360, 403, 471, 483, 670,
729, 842, 843, 941, 971

Here, $n = 14 = \text{even}$

Median value = avg. of $(\frac{n}{2})^{\text{th}}$ & $(\frac{n+1}{2})^{\text{th}}$ items.

$$= (\frac{14}{2}) \quad \& \quad (\frac{14}{2} + 1)$$

= 7th & 8th items in series.

$$= \frac{471 + 483}{2}$$

$$= \frac{954}{2}$$

Median value = 477

[II] CASE : DISCRETE SERIES :

- Arrange the data in ascending / descending order.

(a) Prepare the column of cumulative frequency.

(b) Calculated $\left(\frac{n}{2}\right)$ or $\left(\frac{n+1}{2}\right)$

(c) Find out the value which is just more than $\left(\frac{n}{2}\right)$ or $\left(\frac{n+1}{2}\right)$.

(d) Determine the corresponding x value. This variate value is the median value.

Q: From the data of height of 100 persons given below. Calculate the median value if the variate

Heights (x)	58	60	61	62	63	64	65	66	68	70
No. of persons (f)	4	6	5	10	20	22	24	6	2	1

Solⁿ

Heights (x)	Persons (f)	C.f.
58	4	4
60	6	10
61	5	15
62	10	25
63	20	45
64	22	67
65	24	91
66	6	97
68	2	99
70	1	100

Here, $n = 100 = \text{even}$

$$= \frac{n}{2} = \frac{100}{2} = \boxed{50}$$

- The cumulative frequency just above 50 is 67.

- The value of x corresponding to cumulative frequency 67 is $\boxed{64}$.

$\boxed{\text{Median Height} = 64}$

[III] CASE : CONTINUOUS FREQUENCY DISTRIBUTION :-

- Prepare the column of cumulative frequency distribution.
- Find out the value of $n/2$.
- Find out cumulative frequency just greater than $n/2$.
- The class corresponding to this cumulative frequency is which the median value lies.
- The median value is determined by using the formula \downarrow

$$\text{Median} = L + \frac{n/2 - C_p}{f_m} \times i$$

where,

L = Lower limit of the median class.

n = Total of all frequencies.

C_p = Cumulative frequency preceding the median class.

f_m = frequency corresponding the median class.

i = width of the class interval.

Q: Find out the median value of the number of eggs for the data given below:

No. of eggs	10-12	12-14	14-16	16-18	18-20	20-22	22-24	24-26	26-28
No. of hens	3	11	22	25	32	37	13	10	7

Solⁿ

No. of eggs (x)	No. of hens (f)	C.f
10-12	3	3
12-14	11	14
14-16	22	36
16-18	25	61
18-20	32	93
20-22	37	130
22-24	13	143
24-26	10	153
26-28	7	160

Here, $n = 160$

$$\therefore \frac{n}{2} = \frac{160}{2} = \boxed{80}$$

- The cumulative frequency just greater than 80 is 93.

$$\text{Median class} = 18 - 20$$

$$L = 18, \quad \frac{n}{2} = 80, \quad C_p = 61$$

$$f_m = 32, \quad i = 2$$

$$\text{Median} = L + \frac{\frac{n}{2} - C_p}{f_m} \times i$$

$$= 18 + \frac{80 - 61}{32} \times 2$$

$$= 18 + \frac{19}{32} \times 2$$

$$= 18 + 0.5937 \times 2$$

$$= 18 + 1.187$$

$$\therefore \text{Median} = 19.18$$

Q: Calculate median from the data given below:

Marks below	10	20	30	40	50	60	70	80
No. of students	15	35	60	84	96	127	198	250

Sol:

Marks below	C.f	f
0-10	15	15
10-20	35	20
20-30	60	25
30-40	84	24
40-50	96	12
50-60	127	31
60-70	198	71
70-80	250	52

Here, $n = 250$

$$\therefore \frac{n}{2} = \frac{250}{2} = \boxed{125}$$

- The cumulative frequency greater than 125 is 127.

$$L = 50, \quad \frac{n}{2} = 125, \quad C_p = 96$$

$$j = 10, \quad f_m = 31$$

$$\text{Median} = L + \frac{\frac{n}{2} - C_p}{f_m} \times i$$

$$= 50 + \frac{125 - 96}{31} \times 10$$

$$= 50 + \frac{29}{31} \times 10$$

$$= 50 + 0.935 \times 10$$

$$= 50 + 9.354$$

∴ Median = 59.35 marks

Q: Compute the median for the following frequency distribution:

Size	below 10	10-12	12-14	14-16	16-18	18-20
Demand	3	15	27	20	3	2

Soln

Size	Frequency	C.f.
Below 10	3	3
10 - 12	15	18
12 - 14	27	45
14 - 16	20	65
16 - 18	3	68
18 - 20	2	70

Here, $n = 70$

$$\therefore \frac{n}{2} = \frac{70}{2} = \boxed{35}$$

- The C.f. greater than 35 is 45.

$$L = 12, \quad \frac{n}{2} = 35, \quad C_p = 18, \quad f_m = 27, \quad f = 2$$

$$\text{Median} = L + \frac{\frac{n}{2} - C_p}{f_m} \times i$$

$$= 12 + \frac{35 - 18}{27} \times 2$$

$$= 12 + \frac{17}{27} \times 2$$

$$= 12 + 0.6296 \times 2$$

$$= 12 + 1.259$$

$$\therefore \text{Median} = \boxed{13.259}$$

Q: Following table gives the cumulative frequency of the age group of teachers. Find the mean & median of the group.

Age in years	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65	65-70
C.f.	21	40	90	130	146	166	176	186	195	199

Solⁿ → Mean :

Class	Mid-point	f_i	$d = \frac{x-A}{c}$	$f_i d$
20-25	22.5	21	-4	-84
25-30	27.5	19	-3	-57
30-35	32.5	50	-2	-100
35-40	37.5	40	-1	-40
40-45	42.5 = A	16	0	0
45-50	47.5	20	1	20
50-55	52.5	10	2	20
55-60	57.5	10	3	30
60-65	62.5	9	4	36
65-70	67.5	4	5	20
		$\Sigma f = 199$		$\Sigma f_i d = 155$

$$\text{Mean} = A + \frac{\Sigma f_i d}{\Sigma f} \times 5$$

$$= 42.5 + \frac{(-155)}{199} \times 5$$

$$= 42.5 + (-0.7770) \times 5$$
$$= 42.5 - 3.894$$

$$\therefore \text{Mean} = 38.81$$

→ Median :-

Here, $n = 199$

$$\therefore \frac{n}{2} = \frac{199}{2} = 99.5$$

$L = 35$, $n/2 = 99.5$, $C_p = 90$, $f_m = 40$
 $i = 5$

$$\text{Median} = L + \frac{n/2 - C_p}{f_m} \times i$$

$$= 35 + \frac{99.5 - 90}{40} \times 5$$

$$= 35 + \frac{9.5}{40} \times 5$$

$$= 35 + 0.2375 \times 5$$

$$= 35 + 1.1875$$

$$\therefore \text{Median} = 36.187$$

Q: Find the missing frequency of the following frequency distribution, if the median is 37.5.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	3	?	8	12	10	7	5

Sol:

Class	Frequency	C.f
0-10	3	3
10-20	f_1	$3 + f_1$
20-30	8	$11 + f_1$
30-40	12	$23 + f_1$
40-50	10	$33 + f_1$
50-60	7	$40 + f_1$
60-70	5	$45 + f_1$

Here, $n = 45 + f_1$, $\frac{n}{2} = \frac{45 + f_1}{2}$

Median = 37.5, $L = 30$, $f_m = 12$

$C_p = 11 + f_1$, $i = 10$

$$\text{Median} = L + \frac{\frac{n}{2} - C_p}{f_m} \times i$$

$$\therefore 37.5 = 30 + \frac{45 + f_1}{2} - (11 + f_1) \times 10$$

12

$$\therefore 37.5 = 30 + \frac{45 + f_1 - (11 + f_1)}{2} \times 10$$

$$\therefore 37.5 - 30 = \frac{(45 + f_1) - (22 + f_1)}{24} \times 10$$

$$\therefore 7.5 = \frac{23 + f_1}{24} \times 10$$

$$\therefore 23 + f_1 = \frac{24 \times 7.5}{10}$$

$$\therefore 23 + f_1 = 18$$

$$\therefore f_1 = 23 - 18$$

\therefore $f_1 = 5$ \rightarrow Missing frequency.

Q: Find the median from the following data.

Class	0-30	30-60	60-90	90-120	120-150	150-170
Frequency	8	13	22	27	18	7

Soln:

Class	Frequency	C.f
0-30	8	8
30-60	13	21
60-90	22	43
90-120	27	70
120-150	18	88
150-180	7	95

Here, $n = 95$

$$\therefore \frac{n}{2} = \frac{95}{2} = 47.5$$

$$\text{Median} = L + \frac{\frac{n}{2} - C_p}{f_m} \times i$$

$$= 90 + \frac{47.5 - 43}{27} \times 30$$

$$= 90 + \frac{4.5}{27} \times 30$$

$$= 90 + 0.166 \times 30$$

$$= 90 + 5$$

$$\therefore \text{Median} = 95$$

MODE

* CALCULATION OF MODE :-

[I] CASE : INDIVIDUAL OBSERVATION

- In case of individual observation, modal value can be found out by inspection.
- In case of ungrouped data/individual observations, we count the repetition of every item.
- The value which occurs maximum number of times is the mode of the distribution.

Q: Find the mode of the following values of a group of students.

51, 53, 52, 51, 54, 53, 50, 54, 55,
53, 54, 55, 56 & 54.

Sol: Rearranging the given values in the form of frequency table.

Values (x)	Frequency (f)
50	1
51	2
52	1
53	3
54	4
55	2
56	1

- Since, the maximum frequency is 4, the corresponding value of the variate is 54.

So, mode value = 54.

[II] CASE : DISCRETE SERIES :-

- In certain cases, where 2 frequencies in the data are highest or higher than the rest or the 2 values are very near to each other, then little care will be needed to decide the mode.

- Actually, that frequency shall be taken as modal frequency which has bulk of them in its neighbourhood.

[M] CASE : CONTINUOUS SERIES

$$\text{Mode} = L + \frac{f_m - f_p}{2f_m - f_p - f_1} \times i$$

where,

L = lower limit of modal class

f_m = frequency of the modal class

f_p = frequency preceeding the modal class.

f_1 = frequency following the modal class.

i = width of class interval.

Q: Compute the mode from the table given below for the heights of the students with the corresponding frequencies.

Heights (inches)	Frequency
57-58	17
58-59	27
59-60	74
60-61	19
61-62	22
62-63	43
63-64	37
64-65	14
65-66	8
66-67	12
67-68	5
68-69	3

- Here, the maximum frequency is 43.

The corresponding modal class is 62-63.

We know,

$$\text{Mode} = L + \frac{f_m - f_p}{2f_m - f_p - f_i} \times i$$

$$L = 62, f_m = 43, f_p = 22, \\ f_1 = 37, i = 1$$

$$\text{Mode} = 62 + \frac{43 - 22}{2 \times 43 - 22 - 37} \times 1$$

$$= 62 + \frac{21}{27}$$

$$= 62 + 0.777$$

$$\therefore \boxed{\text{Mode} = 62.78 \text{ inches}}$$

Q: Compute the A.M, median & mode of the heights of 15 patients.

The heights are 61, 62, 63, 61, 63, 64, 64, 64, 60, 65, 63, 64, 65, 66, 64.

Solⁿ: $A.M = \frac{\sum x_i}{n}$

Here, $n = 15$, $\sum x_i = 949$

$$A.M = \frac{949}{15}$$

$$= \boxed{63.27}$$

Now, we arrange the given observations in ascending order of magnitudes.

60, 61, 61, 62, 63, 63, 63, 64, 64, 64,
64, 64, 65, 65, 66.

$$\begin{aligned} \text{Median} &= \left(\frac{n+1}{2} \right) \\ &= \left(\frac{15+1}{2} \right) \\ &= 8^{\text{th}} \text{ value} \\ &= \boxed{64} \end{aligned}$$

Mode is the most frequently occurring value.

$$\text{Mode} = \boxed{64}$$

Q: Find mean, median & mode for the following data.

x	1	2	3	4	5	6	7
f	7	12	21	27	19	11	3

Solⁿ

MEAN:

x	f	fx
1	7	7
2	12	24
3	21	63
4	27	108
5	19	95
6	11	66
7	13	21
	$\Sigma f = 100$	$\Sigma fx = 384$

$$A.M = \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{384}{100}$$

$$\therefore \bar{x} = 3.84$$

→ MEDIAN:

x	f	C.f
1	7	7
2	12	19
3	21	40
4	27	67
5	19	86
6	11	97
7	3	100

Median = $\frac{n}{2} = \frac{100}{2} = 50^{\text{th}}$ value.

The C.f. just above 50 is 67

The value of x corresponding to cumulative frequency 67 is 4.

Median = 4

→ MODE :=

Maximum frequency 27 occurs against $x = 4$.

Mode = 4

Q. Calculate the mode of following data.

Class interval	Frequency
0 - 10	10
10 - 20	14
20 - 30	19
30 - 40	17
40 - 50	13

Maximum frequency = 19
 $f_m = 19$, $L = 20$, $f_p = 14$
 $f_1 = 17$, $i = 10$

$$\text{Mode} = L + \frac{f_m - f_p}{2f_m - f_p - f_1} \times i$$

$$= 20 + \frac{19 - 14}{2 \times 19 - 14 - 17} \times 10$$

$$= 20 + \frac{5}{7} \times 10$$

$$= 20 + 0.714 \times 10$$

$$= 20 + 7.142$$

$$\therefore \boxed{\text{Mode} = 27.14}$$

Q: The following data is the result of 400 HIV positive men in a reported age group of India. Calculate the mode from the given data.

Age limit	Frequency
20 - 24	2
25 - 29	85
30 - 34	135
35 - 39	91
40 - 44	57
45 - 49	20
50 - 54	10

Maximum frequency (f_m) = 135

$L = 30$, $f_p = 85$, $f_1 = 91$

$i = 4$

$$\text{Mode} = L + \frac{f_m - f_p}{2f_m - f_p - f_1} \times i$$

$$= 30 + \frac{135 - 85}{2 \times 135 - 85 - 91} \times 4$$

$$= 30 + \frac{50}{94} \times 4$$

$$= 30 + 0.5319 \times 4$$

$$= 30 + 2.1276$$

\therefore Mode = 32.127

Q: Find the mode for the following frequency distribution.

Size	Demand
Below 10	3
10 - 12	15
12 - 14	27
14 - 16	20
16 - 18	3
18 - 20	2

Maximum frequency (f_m) = 27
 $L = 12$, $f_p = 15$, $f_1 = 20$, $i = 2$

$$\text{Mode} = L + \frac{f_m - f_p}{2f_m - f_p - f_1} \times i$$

$$= 12 + \frac{27 - 15}{2 \times 27 - 15 - 20} \times 2$$

$$= 12 + \frac{12}{19} \times 2$$

$$= 12 + 0.6315 \times 2$$

$$= 12 + 1.263$$

\therefore $\text{Mode} = 13.26$