4th sem
B.Pharm

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Page No. 1
Date

Ch-: Measure of central tendency
(Average)

⇒ Measure of central tendency are Also usually called as the

Averages

⇒ By Calculating the measure of central tendency, we can find a single value to Represent whole data

it Also Help to compare the value of two or more groups.

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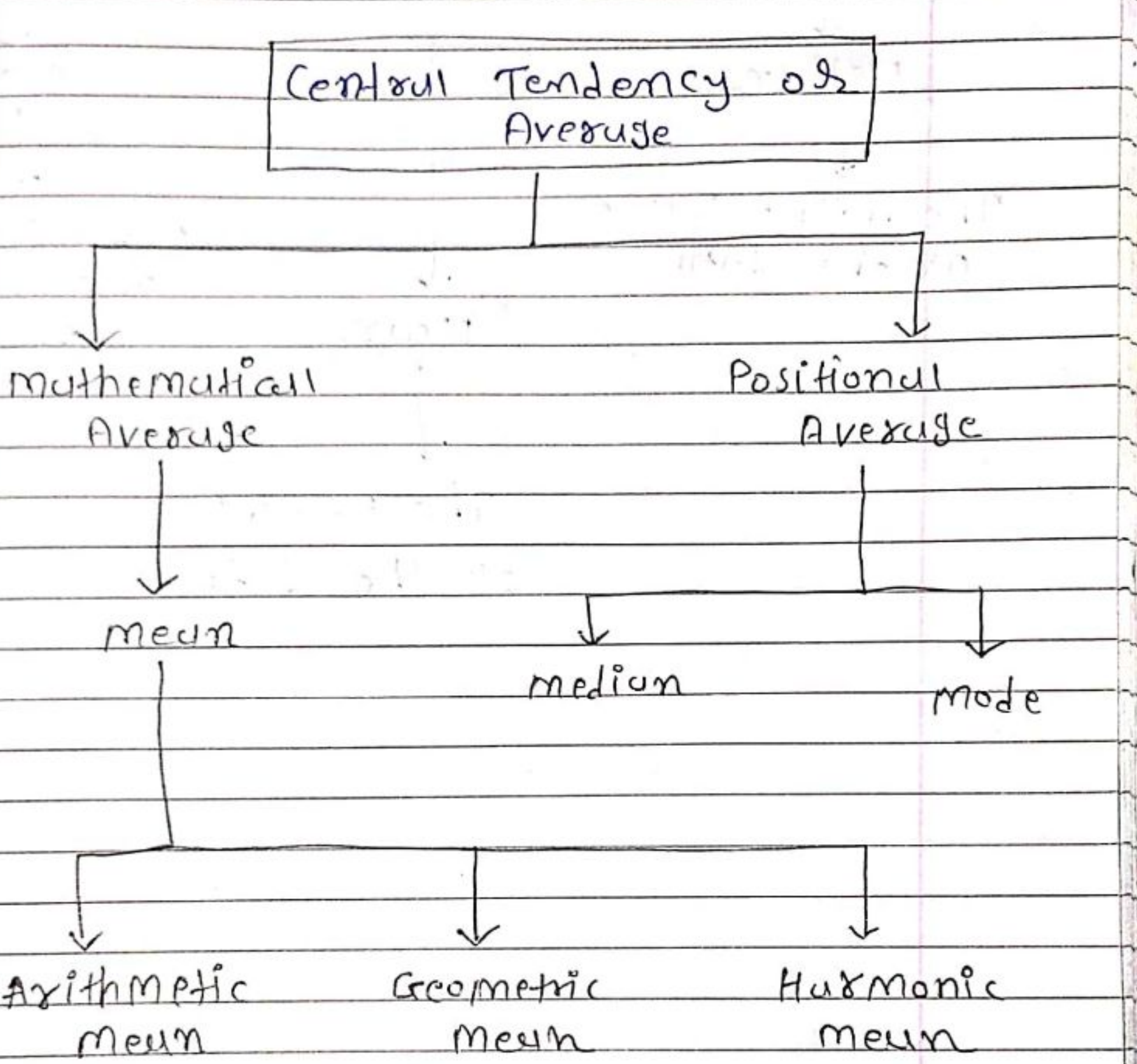
Page No. 1
Date

* Types of measure of central Tendency



Measure of Central Tendency

- Median
  - The middle value of the data
- Mean
  - The Average of the data
- Mode
  - most commonly occurring value

- Mean
  - Arithmetic mean
    - series data
    - Ungrouped data
      - Direct method
      - change of origin
      - step Deviation method
        - change of origin and scale
  - Geometric mean
    - Ungrouped frequency distribution (discrete series
  - Harmonic mean
    - grouped frequency distribution (continuous series]

Median

(1) case-: Single series

(2) case-: Discrete series

(3) case-: continuous Frequency Distribution

Mode

(1) case-: individual observation or in case of Discrete distribution

(2) case-: Discrete series

(3) case-: continuous series

* Mean

* All formula for Mean ↓

(1) Arithmetic mean ↓

for series

$$\bar{x} = \frac{\Sigma x_i}{n}$$

where n = Total Number of observation

x_i = sum of total Number of observation

(2) Ungrouped Frequency distribution
=> (or [Discrete series])

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

(3) continuous series (grouped frequency distribution)

(4) Direct Method ↓

$$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$

(b) step-deviation method ↓ (short cut method)

(i) change of origin ↓

(a) for ungroup data ↓

$$\bar{y} = \frac{y_1 + y_2 + \dots + y_n}{n}$$

(b) for ungrouped frequency distribution

$$\bar{y} = \frac{1}{n} \Sigma fy$$

Required A.M $= \bar{x} = A + \bar{y}$

* short-cut method ↓

$$X = A + \frac{\Sigma fdx}{\Sigma f}$$

(c) for grouped frequency distribution

$\bar{y} = \frac{1}{n} \Sigma f_i y_i$.

Required A.M $\bar{x} = A + \bar{y}$

(iii) change of origin and scale both

$y_i = \frac{x_i - A}{d}$ $\bar{y} = \frac{1}{n} \Sigma f_i y_i$

Required = A.M = $\bar{x} = A + d\bar{y}$

(4) Geometric mean ↓

$$G.M = \sqrt{x_1 \cdot x_2 \cdot x_3 \cdot \dots x_n}$$

(5) Harmonic Mean ↓

$$\frac{1}{H} = \frac{1}{n}\left[\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}\right)$$

$= \frac{1}{n}$ (Reciprocal of x_i)

$$H = \frac{n}{\Sigma \text{Reciprocal of } x_i}$$

(11) Find the A.M mean of
3, 6, 5, ۲, 6, 11, 13, 19, 12 and 7.

Ans

$$A.M = \bar{x} = \frac{\Sigma x_i}{n}$$

$$= \frac{3+6+5+۲+6+11+13+19+12+7}{10}$$

$$= \frac{۹۰}{10}$$

$$\bar{x} = ۹$$

(2) The labeled potency of a tablet dosage form is 100mg. Ten individual tablets are assayed According to a quality control specification. The Result are 101.6, 102, 99.6, 104.7, 103.2, 104.6, 100.5, 106.3, 100.7, 105.2 Find the Average Dosage

Ans

$$A.M = \bar{X} = \frac{\Sigma Xi}{n}$$

$$\bar{X} = \frac{101.6 + 102 + 99.6 + 104.7 + 103.2 + 104.6 + 100.5 + 106.3 + 100.7 + 105.2}{10}$$

$$\bar{X} = \frac{1029}{10}$$

$$\boxed{\bar{X} = 102.9}$$

(3) Find out the A.M mean of
4, 14, 10, 22, 76, 50, 35, 20

Ans

$$A.M = \bar{x} = \frac{\Sigma x_i}{n}$$

$$\bar{x} = \frac{4+14+10+22+76+50+35+20}{8}$$

$$= \frac{231}{8}$$

$$\boxed{\bar{x} = 28.87}$$

(4) The mean weight of a patient in a group of six patients is 119. The individual weights of five of them are 115, 109, 129, 117 and 114. What is weight of the sixth patient?

$$\bar{X} = \frac{\Sigma x_i}{n}$$

$$119 = \frac{109 + 115 + 129 + 117 + 114 + x}{6}$$

$$119 = \frac{584 + x}{6}$$

$$119 \times 6 = 584 + x$$

$$714 = 584 + x$$

$$x = 714 - 584$$

$x = 130$ weight of six Patient

(5) Calculate average wage paid per worker from the data given below

Wages (Rx) (X)	500	600	700	800	900	1000	1100
Workers (f)	1	3	5	7	6	2	1

Ans

Wages X	F	Fx
500	1	500
600	3	1800
700	5	3500
800	7	5600
900	6	5400
1000	2	2000
1100	1	1100
	$\Sigma f = 25$	$\Sigma fx = 19900$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{19900}{25}$$

$$\bar{x} = 796$$

(6) The following frequency distribution shows the Number of days of confinement of Patients after Delivery. find the means days of confinement

Day of confinement x	6	7	8	9	10
No of Patients f	7	6	6	4	2

And

x	f	fx
6	7	42
7	6	42
8	8	48
9	4	36
10	2	20
	$\Sigma f = 25$	$\Sigma fx = 188$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{188}{25}$$

$$\boxed{\bar{x} = 7.52}$$

(7) Find the A.M by direct Method of the following data

size (c)	3-5	5-7	7-9	9-11	11-13
frequency (f)	14	16	25	22	12

Ans

size of class	Mid Point (x)	frequency (f)	fx
3-5	4	14	56
5-7	6	16	96
7-9	8	25	200
9-11	10	22	220
11-13	12	12	144
		$n = 89$	$\Sigma fx = 716$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{716}{89} = \boxed{8.04}$$

(Y) Compute the Avg. Marks of students using the following data ↓

Marks	0-10	10-20	20-30	30-40	40-50
No of Students	3	19	32	17	4

Ans

Marks	Mid Point (x)	frequency (f)	fx
0-10	5	3	15
10-20	15	19	285
20-30	25	32	800
30-40	35	17	595
40-50	45	4	180
		$\Sigma f = 75$	$\Sigma fx = 1875$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1875}{75} = \boxed{25}$$

(9) $x = 3, 6, 5, 8, 6, 11, 13, 19, 12, 7$ and calculate A.M by change of origin method

Ans

$$y_i = x - 3$$

$$y = 0, 3, 2, 5, 3, 8, 10, 16, 9, 4$$

$$\bar{y} = \frac{0+3+2+5+3+8+10+16+9+4}{10}$$

$$= \frac{60}{10}$$

$$\boxed{\bar{y} = 6}$$

Required mean $\bar{x} = 3 + \bar{y} = 3 + 6 = \boxed{9}$

(10) $\bar{X} = 2, 4, 6, 10, 14, 24, 34, 40$

calculated A.M by change of origin method

Ans

$y_i = x - 2$

$y = 0, 2, 4, 8, 12, 22, 32, 38$

$$\bar{y} = \frac{0+2+4+8+12+22+32+38}{8}$$

$$\bar{y} = \frac{118}{8}$$

$\bar{y} = 14.75$

Required mean $= \bar{y} + 2$

$= 14.75 + 2$

$= 16.75$

(11)

Wages (Rs) x	500	600	700	800	900	1000	1100
No of workers (f)	1	3	5	7	6	2	1

Calculate the A.M by change of origin Method.

Ans

Wages (Rs) x	No of workers (f)	deviation $y = x - 800$	fy
500	1	-300	-300
600	3	-200	-600
700	5	-100	-500
800	7	0	0
900	6	100	600
1000	2	200	400
1100	1	300	300
	$\Sigma f = 25$		-100

$$\bar{y} = \frac{\Sigma f y}{n}$$

$$= \frac{-100}{25}$$

$$\boxed{\bar{y} = -4}$$

Required Mean $= 800 + \bar{y}$

$= 800 - 4$

$\boxed{= 796}$

Q12)

Class intervals	3-5	5-7	7-9	9-11	11-13
freq. (f)	14	16	25	22	12

Calculated A.M by change of origin Method

Ans

class intervals	mid point x_i	(f)	$y_i = x_i - 8$	$f_i y_i$
3-5	4	14	-4	-56
5-7	6	16	-2	-32
7-9	8	25	0	0
9-11	10	22	2	44
11-13	12	12	4	48
		$n = 89$		+4

$$\bar{y} = \frac{\Sigma f y}{n}$$

$$= \frac{4}{89}$$

$$\boxed{\bar{y} = 0.0449}$$

Required mean $= 8 + \bar{y}$

$= 8 + 0.0449$

$\boxed{= 8.045}$

(13)

Class (x)	10	20	30	40	50	60
frequency (f)	3	2	5	10	11	4

Calculate the mean by change of origin or short cut method

Ans

class (x)	frequency (f)	$dx_i = x - A$	fdx_i
10	3	-30	-90
20	2	-20	-40
30	5	-10	-50
A = 40	10	0	0
50	11	10	110
60	8	20	160
	$\Sigma f = 39$		$\Sigma fdx_i = 90$

$$\bar{X} = A + \frac{\Sigma f d_i c_i}{\Sigma f}$$

$$= 40 + \frac{90}{34}$$

$$= 40 + 2.31$$

$$\boxed{\bar{X} = 42.31}$$

(12) The following table shows a distribution of children per family of 191 families. Find the average number of children per family by short cut method.

No of children	0	1	2	3	4	5	6	7	8
No of Families	25	16	36	48	30	16	15	3	2

Ans

No of children (x)	(f)	$dx = x - A$	fdx
0	25	-4	-100
1	16	-3	-48
2	36	-2	-72
3	48	-1	-48
4 = A	30	0	0
5	16	1	16
6	15	2	30
7	3	3	9
8	2	4	8
	$\Sigma f = 191$		$\Sigma fdx = -205$

$$\bar{x} = A + \frac{\Sigma fdx}{\Sigma f}$$

$$= 4 + \frac{(-205)}{191}$$

$$= 4 + (-1.07)$$

$= 4 - 1.07$

$\bar{X} = 2.93$

[15] The following table shows a frequency distribution of a Age of Patients examined on a particular day in the OPD of a Hospital. find the Average age of the Patients by short cut method.

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No of Patients	23	57	138	277	105	74	21

Ans

Age in years	Mid Point (x_i)	f_i	$y_i = x_i - 35$	$f_i y_i$
0-10	5	23	-30	-690
10-20	15	57	-20	-1140
20-30	25	138	-10	-1380
30-40	35	277	0	0
40-50	45	105	10	1050
50-60	55	74	20	1480
60-70	65	21	30	630
		$\Sigma f = 695$		$\Sigma f_i y_i = -50$

$$\bar{y} = \frac{\Sigma f_i y_i}{\Sigma f}$$

$$= \frac{(-50)}{695}$$

$= -0.0719$

Required Mean $= 35 - 0.0714$

$$\bar{X}_1 = 34.92$$

(16). Given below is the distribution of 140 candidates obtaining mark X or Higher in a certain examination

X	10	20	30	40	50	60	70	80	90	100
cum freq (f)	140	133	116	100	75	45	25	9	2	0

calculate the mean of the marks obtained by short cut method

Ans)

Assume → $c = 10$

Class	Mid Point (x_i)	f_i	$d = \frac{x_i - A}{c}$	$f_i d$
10-20	15	140-133 = 7	-4	-28
20-30	25	133-118 = 15	-3	-45
30-40	35	118-100 = 18	-2	-36
40-50	48	100-75 = 25	-1	-25
50-60	A = 55	75-48 = 30	0	0
60-70	65	48-25 = 20	1	20
70-80	75	25-9 = 16	2	32
80-90	85	9-2 = 7	3	21
90-100	95	2-0 = 2	4	8
		$\Sigma f = 140$		$\Sigma f_i d = -53$

$$\bar{X} = A + \frac{\Sigma fid}{\Sigma f} \times C$$

$$= 55 + \frac{(-53)}{140} \times 10$$

$$= 55 + (-0.37) \times 10$$

$$= 55 + (-3.7)$$

$$= 55 - 3.77$$

$$= 51.23$$

(17) Find the mean weight of 470 infants born in a Hospital, in one year, from the following table:

Weight of Infants (kg)	2-2.4	2.5-2.9	3-3.4	3.5-3.9	4-4.4	4.5-4.5
No of Infants	17	97	147	135	28	6

$c = 0.1$

Class x	Mid point x_i	f_i	$d = \frac{x - A}{c}$	$f_i d$
2 - 2.4	2.2	17	-10	-170
2.5 - 2.9	2.7	47	-5	-475
3 - 3.4	A = 3.2	187	0	0
3.5 - 3.9	3.7	135	5	675
4 - 4.4	4.2	28	10	280
4.5 - 4.9	4.7	6	15	90
		$\Sigma f_i = 470$		$\Sigma f_i d = 390$

$$\bar{x} = A + \frac{\Sigma f_i d}{\Sigma f} \times c$$

$$= 3.2 + \frac{390}{470} \times 0.1$$

$$= 3.2 + 0.82 \times 0.1$$

$= 3.2 + 0.0r2$

$\boxed{\bar{x} = 3.2r2}$

(18) Following table gives the cumulative frequency distribution of the ages of 200 teachers. Find the mean of the distribution

Age in years	cumulative frequency	Age in years	cumulative frequency
20-25	22	45-50	167
25-30	41	55-55	177
30-35	91	50-60	187
35-40	132	60-65	199
40-50	147	65-70	200

Solution

Class	Mid Point x_i	f_i	$d = \frac{x_i - A}{c}$	$f_i d$
20-25	22.5	22	-4	-88
25-30	27.5	19	-3	-57
30-35	32.5	50	-2	-100
35-40	37.5	41	-1	-41
40-45	A = 42.5	15	0	0
45-50	47.5	20	1	20
50-55	52.5	10	2	20
55-60	57.5	10	3	30
60-65	62.5	12	4	48
65-70	67.5	1	5	5
		$\Sigma f = 20$		$\Sigma f_i d = -163$

$$\bar{x} = A + \frac{\Sigma fid}{\Sigma f} \times c$$

$$= 42.5 + \frac{(-163)}{200} \times 5$$

$$= 42.5 + (-0.815) \times 5$$

$$= 42.5 - 4.075$$

$$\boxed{\bar{x} = 38.425}$$

[19] (f)

class interval	3-5	5-7	7-9	9-11	11-13
	14	16	28	22	12

Calculate the A.M by change of origin and scale.

Ans

class interval	Mid point x_i	f_i	$d = \frac{x_i - A}{c}$	$f_i d$
3-5	4	14	-2	-28
5-7	6	16	-1	-16
7-9	$\boxed{8} = A$	25	0	0
9-11	10	22	1	22
11-13	12	12	2	24
		$\Sigma f_i = 89$		$\Sigma f_i d = 2$

$$\bar{x} = A + \frac{\Sigma f_i d}{\Sigma f_i} \times c$$

$$= 8 + \frac{2}{89} \times 2$$

$$= 8 + 0.022 \times 2$$

$$= 8 + 0.044$$

$$\boxed{\bar{x} = 8.044}$$

* Geometric mean

Q) 15, 5415

$$G.M = \sqrt[n]{x_1 \cdot x_2}$$

$$= \sqrt{15 \times 5415}$$

$$= \sqrt{81255}$$

G.M = 285

Q) 5, 6, 25, 5/6

$$G.M = \sqrt[4]{x_1 \cdot x_2 \cdot x_3 \cdot x_4}$$

$$= \sqrt[4]{5 \cdot \cancel{6} \cdot 25 \cdot \frac{5}{\cancel{6}}}$$

$$= \sqrt[4]{5 \times 25 \times 5}$$

$$= \sqrt[4]{5^4}$$

$$= 5$$

* Harmonic Mean.

ex: Find the Harmonic mean of $5/2$, 5, 10 and $10/3$

Soln:

$$H.M = \frac{n}{\sum \frac{1}{x_i}}$$

$$= \frac{n}{\frac{2}{5} + \frac{1}{5} + \frac{1}{10} + \frac{3}{10}}$$

$$= \frac{4}{\frac{2}{5} + \frac{1}{5} + \frac{1}{10} + \frac{3}{10}}$$

$$= \frac{4}{\frac{4}{10} + \frac{2}{10} + \frac{1}{10} + \frac{3}{10}}$$

$$= \frac{4}{1\frac{10}{10}}$$

$$= 4/1$$

$$\boxed{H.M = 4}$$

* Median

* formula

(1) single series

→ Rearrange the data in ascending / descending order

→ if series is odd then formula

$$= \left(\frac{n+1}{2}\right)$$

→ if series is even then formula

$$= \left(\frac{n}{2}\right) \text{ and } \left(\frac{n+1}{2}\right)$$

(17) Find the mean value of the data given below:-

Income (Rs.) 1100, 1150, 1040, 1120, 1200, 1400

1400

Sol^n Rearrange the data in Ascending order thus:-

1040, 1100, 1120, 1150, 1190, 1200, 1400

Here = $n = 7$ = odd

Median value = $\frac{n+1}{2}$

= $\frac{7+1}{2}$

= 4^{th} in order

= 1150

(2) Find out the medium value of the following data

10, 9, 11, 12, 6, 8, 13 and 14

Solution

arranged the order in ascending order ↓

6, 8, 9, 10, 11, 12, 13, 14

Here $n = 8$ = even

Median value = average of $\left(\frac{n}{2}\right)$ item and $\left(\frac{n+1}{2}\right)$ item

$= \left(\frac{8}{2}\right)$ and $\left(\frac{8+1}{2}\right)$

$= 4^{th}$ and 5^{th}

$= \frac{10+11}{2} = \boxed{10.5}$

(37) The following figures show incubation periods of 9 Polio cases. Find the medium of the date

23, 19, 21, 17, 22, 18, 20, 24, 16

Ans Rearranged the date in Ascending order ↓

16, 17, 18, 19, 20, 21, 22, 23, 24

Here $n = 9 =$ odd

Medium value $= \left(\frac{n+1}{2}\right)$

$= \frac{9+1}{2}$

$= 5^{th}$ items

$= 20$

(4) Find the medium of the data set

102, 403, 729, 843, 920, 360, 842,
941, 357, 483, 207, 670, 471, 109

Ans

arranged in data Ascending order ↓

102, 109, 207, 357, 360, 403,
471, 483, 670, 729, 842, 843,
941, 971

Here = $n = 14$

Medium value = average of $\left(\frac{n}{2}\right)$ items and $\left(\frac{n}{2}+1\right)$ items

$= \left(\frac{14}{2}\right)\left(\frac{14}{2}+1\right)$

$= (7)^{th}(8)^{th}$ items in series

$$= \frac{491 + 483}{2}$$

$$= \frac{954}{2}$$

$$= 477$$

(2) Discrete series ↓

(a) Arrange the data in Ascending/ descending order

(b) Prepare the column of cumulative frequency

(c) Calculated $\left(\frac{n}{2}\right)$ or $\left(\frac{n}{2}+1\right]$

(d) Find out the value which is just more than $\left(\frac{n}{2}\right)$ or $\left(\frac{n}{2}+1\right)$

(e) Determine the corresponding x value. This variate value is the medium value

ex:-1 From the data of Heights of 100 persons given below calculate the medium value if the variate

Heights x	58	60	61	62	63	64	65	66	67	70
No of persons (f)	4	6	5	10	20	22	24	6	2	1

Ans

Heights (x)	persons (f)	cumulative foe (c.f)
58	4	4
60	6	10
61	5	15
62	10	25
63	20	45
64	22	67
65	24	91
66	6	97
67	2	99
70	1	100

Here Σ 100 = even

$= \frac{n}{2} = \frac{100}{2} = \boxed{50}$

The cumulative frequency just above 50 is ↴

67

The value of X corresponding to cumulative frequency 67 is [64]

Medium Height = [64]

[3] Continuous frequency Distribution

(a) Prepare the column of cumulative frequency distribution.

(b) Find out the value of $n/2$.

(c) Find out cumulative frequency just greater than $n/2$.

(d) The class corresponding to this cumulative frequency is which the medium value lies.

(e) The medium value is determined by using the formula

$$\text{Median} = L + \frac{\frac{n}{2} - c_p}{f_m} \times i$$

where :-

L = Lower limit of the medium class

n = Total of all frequencies

c_p = cumulative frequency preceding the medium class.

f_m = frequency corresponding the median class

i = width of the class interval

ex(1) Find out the Medium value of the Number of eggs for the data given below:

No of eggs	10-12	12-14	14-16	16-18	18-20	20-22	22-24	26 24	28 26
No of Hens	3	11	22	25	32	37	13	10	7

Ans

No. of eggs	No. of Hens	Cumulative frequency
10-12	3	3
12-14	11	14
14-16	22	36
16-18	25	61
18-20	32	93
20-22	37	130
22-24	13	143
24-26	10	153
26-28	7	160

Here $160 = n$

$$\frac{n}{2} = \frac{160}{2} = \boxed{80}$$

⇒ The cumulative frequency just greater than 80 is 93

Medium class = 18-20

$L = 18 \quad \frac{n}{2} = 80, \quad C_p = 61$

$f_m = 32 \quad i = 2$

$$\text{median} = L + \frac{\frac{n}{2} - C_p}{f_m} \times i$$

$$= 18 + \frac{80 - 61}{32} \times 2$$

$$= 18 + \frac{19}{32} \times 2$$

$= 18 + 0.5937 \times 2$

$= 18 + 1.187$

$= 19.18$

ex Calculate median from the data given below

Marks below	10	20	30	40	50	60	70	80
No of students	15	35	60	84	96	127	191	250

Ans

Marks below	c. frencery	~~Cumulative~~ frencery
0 - 10	15	15
10 - 20	35	20
20 - 30	60	25
30 - 40	84	24
40 - 50	96	12
50 - 60	127	31
60 - 70	191	71
70 - 80	250	52

Here $n = 250$

$$\frac{n}{2} = \frac{250}{2} = \boxed{125}$$

$\Rightarrow$ The cumulative frequency greater than 125 is 127

$L = 50 \qquad \frac{n}{2} = 125 \qquad Cp = 96$

$i = 10 \qquad fm = 31$

$$\text{Median} = L + \frac{\frac{n}{2} - Cp}{fm} \times i$$

$$= 50 + \frac{125 - 96}{31} \cdot 10$$

$$= 50 + \frac{29}{31} \times 10$$

$= 50 + 0.935 \times 10$

$= 50 + 9.354$

$= 59.35$ Marks

ex Compute the medium for the following frequency distribution

size	below 10	10-12	12-14	14-16	16-18	18-20
Demand	3	15	27	20	3	2

Ans

size	frequency	c.f.
below 10	3	3
10-12	15	18
12-14	27	45
14-16	20	65
16-18	3	68
18-20	2	70

Here $n = 70$

$$= \frac{n}{2} = \frac{70}{2} = \boxed{35}$$

The c.f greather thum 35 is 7.

45

$L = 12 \quad \frac{n}{2} = 35 \quad cp = 17 \quad fm = 27$

$i = 2$

$$\text{median} = L + \frac{\frac{n}{2} - cp}{fm} \cdot i$$

$$= 12 + \frac{35 - 17}{27} \cdot 2$$

$$= 12 + \frac{17}{27} \cdot 2$$

$$= 12 + 0.6296 \cdot 2$$

$= 12 + 1.259$

$= 13.259$

ex Following table gives the cumulative frequency of the age group of teachers. find the mean and medium of the group.

Age in years	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65	65-70
c.f	21	40	90	130	146	166	176	186	195	199

Ans find mean I

class	mid point (x_i)	(f_i)	$d = \frac{x-A}{c}$	$f_i d$
20-25	22.5	21	-4	-84
25-30	27.5	19	-3	-57
30-35	32.5	50	-2	-100
35-40	37.5	40	-1	-40
40-45	42.5 = A	16	0	0
45-50	47.5	20	1	20
50-55	52.5	10	2	20
55-60	57.5	10	3	30
60-65	62.5	9	4	36
65-70	67.5	4	5	20
		$\Sigma f = 199$		$\Sigma f_i d = -155$

$$\text{mean} = A + \frac{\Sigma fid}{\Sigma f} \times c$$

$$= 42.5 + \frac{(-155)}{199} \times 5$$

$$= 42.5 + (-0.7788) \times 5$$

$$= 42.5 - 3.894$$

$$= 38.66$$

Median $n = 199$

$$= \frac{n}{2} = \frac{199}{2} = 99.5$$

$L = 35 \quad \frac{n}{2} = 99.5 \quad c_p = 90 \quad i = 5$

$fm = 40$

$$\text{Median} = L + \frac{\frac{n}{2} - C_p}{f_m} \cdot i$$

$$= 35 + \frac{99.5 - 90}{40} \cdot 5$$

$$= 35 + \frac{9.5}{40} \cdot 5$$

$$= 35 + 0.2375 \cdot 5$$

$$= 35 + 1.1875$$

$$\boxed{= 36.187}$$

ex = Find the missing frequency of the following frequency distribution if the median is 37.5

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
frequency	3	9	8	12	10	7	5

Class	frequency	cumulative frequency
0-10	3	3
10-20	f_1	$3+f_1$
20-30	8	$11+f_1$
30-40	12	$23+f_1$
40-50	10	$33+f_1$
50-60	7	$40+f_1$
60-7	5	$45+f_2$

Here $n = 45+f_1$ $\quad \frac{n}{2} = \frac{45+f_1}{2}$

Median = 37.5 $\quad$ class (L) = 30

$f_m = 12$ $\quad cp = 11+f_1$ $\quad i = 10$

$$M = L + \frac{n/2 - cp}{f_m} \times i$$

$$37.5 = 30 + \frac{\frac{45+f_1}{2} - (11+f_1)}{12} \times 10$$

$$37.5 - 30 = \frac{(45+f_1) - (22+f_1)}{24} \times 10$$

$$7.5 = \frac{23+f_1}{24} \times 10$$

$$23+f_1 = \frac{24 \times 7.5}{10}$$

$$23 - f_1 = 18$$

$$\therefore f_1 = 23 - 18$$

$$\boxed{f_1 = 5}$$ missing frequency

ex// Find the median from the following data

class	0-30	30-60	60-90	90-120	120-150	150-180
frequency	8	13	22	27	18	7

Ans//

class	frequency	c.f
0-30	8	8
30-60	13	21
60-90	22	43
90-120	27	70
120-150	18	88
150-180	7	95

Here $= n = 95$

$$\frac{n}{2} = \frac{95}{2} = 47.5$$

$L = 90 \quad Cp = 43 \quad fm = 27$

$i = 30 \quad n/2 = 47.5$

$$\text{Median} = L + \frac{\frac{n}{2} - Cp}{fm} \times i$$

$$= 90 + \frac{47.5 - 43}{27} \cdot 30$$

$$= 90 + \frac{4.5}{27} \cdot 30$$

$$= 90 + 0.166 \cdot 30$$

$$= 90 + 5$$

$$\boxed{= 95}$$

* Mode

(a) individual observation or in case of discrete distribution

⇒ in case of individual observation modal value can be found out by inspection.

⇒ in case of ungrouped data/ individual observations, we count the Repetition of every item.

⇒ The value which occurs maximum number of times is the mode of the distribution.

ex: Find the mode of the following values of a group of students

51, 53, 52, 51, 54, 53, 50, 54, 55, 53, 54, 55, 56 and 54

Ans // Arranging the given values in the form of frequency table.

Values (X)	Frequency (f)
50	1
51	2
52	1
53	3
54	4
55	2
56	1

⇒ Since the maximum frequency is 4 the corresponding value of the variate is 54

So Mode Value = 54

(6) Discrete Series

⇒ in certain cases where two frequencies in the duty are Highest or Higher than the Rest of the two values are very near to each other then little care will be needed to decide the mode

⇒ Actually, that frequency shall be taken as modal frequency which has bulk of them in its neighborhood.

Ex

Number of Pods	Number of std plants	Number of Pods	Number of std plants
50	2	57	32
51	1	58	6
52	4	59	4
53	5	60	3
54	7	61	2
55	31	62	6
56	10	63	2
		64	3
		65	2

⇒ in this data the Highest frequency is 32 and the next Higher frequency is 31.

⇒ Since these two frequency are More or less equal as compared to the other frequency, it becomes Necessary to decide the Mode out of these value of High frequency.

Number of pods	No of plants	Grouping of 2's	Grouping of 2's leaving 1st	Grouping of 3's	Grouping of 3's leaving 1st	Grouping of 3's leaving 1st and 2nd
50	2					
51	1	3				
52	4		5	7		
53	5	9			10	
54	7		12	43		16
55	31	35			48	
56	10		41			73
57	32	42		48		
58	6		38		42	
59	4	10				13
60	3		7	9		
61	2	5			11	
62	6		8			10

63	2	8				
64	3	5	5			
65	2				7	
~~Sd~~	~~5 6 5 7~~	~~5 5 9 60~~				
Variate having max. freq. (x)	57	56, 57	55, 56	56, 57 58	54, 55, 56	55, 56 57

⇒ To decide which variate value is involved in maximum number of times, the following table is made ↓

Groups	Variables				
	54	55	56	57	58
single				✓	
Two's			✓	✓	
Two's leaving 1st		✓	✓		
Three's			✓	✓	✓
Three's leaving 1st	✓		✓		
Three's leaving 1st and 2nd			✓	✓	
Total	1	3	5	4	1

⇒ The variable 56 occurs 5 times.

⇒ So Required mode value ↓

Ans :- 56

[c] continuous series ↓

$$\text{Mode} = L + \frac{fm - f_{-1}}{2fm - f_{-1} - f_1} \times i$$

where ↓

L = Lower limit of the modal class

fm = frequency of the modal class

f_{-1} = frequency preceding the modal class

f_1 = frequency following the modal class

i = width of the class interval

ex Compute the mode from the table given below for the Heights of the students with the corresponding frequencies.

Heights (inches)	frequency
57 - 58	17
58 - 59	27
59 - 60	14
60 - 61	19
61 - 62	22
62 - 63	43
63 - 64	37
64 - 65	14
65 - 66	8
66 - 67	12
67 - 68	5
68 - 69	3

Soln → Here the maximum frequency is ↓

43

The corresponding modal class is 62 - 63.

We know

$$\text{Mode} = L + \frac{f_m - f_{-1}}{2f_m - f_{-1} - f_1} \times i$$

Here $L = 62$ $f_m = 43$, $f_{-1} = 22$

$f_1 = 37$ $i = 1$

$$\text{Mode} = 62 + \frac{43 - 22}{2 \times 43 - 22 - 37} \times 1$$

$$= 62 + \frac{21}{27}$$

$$= 62 + 0.777$$

$$= 62.78 \text{ inches}$$

ex = Compute the A.M, Medium and Mode of the Heights of 15 Patients. The Height are 61, 62, 63, 61, 63, 64, 64, 64, 60, 65, 63, 64, 65, 66, 64

Ans =

$$A.M = \frac{\Sigma x_i}{n}$$

Here $n = 15$ $\Sigma x_i = 949$

$$A.M = \frac{949}{15}$$

$$= 63.27$$

We, Now arrange the given observations in Ascending order of magnitudes

60, 61, 61, 62, 63, 63, 63, 64, 64, 64, 64, 64, 65, 65, 66

$$\text{median} = \left(\frac{n+1}{2}\right)$$

$$= \left(\frac{15+1}{2}\right)$$

$$= 8^{th}. \text{ value}$$

$$= 64$$

mode is the most frequently occurring value

mode = 64

ex find mean, median and mode for the following data

x	1	2	3	4	5	6	7
f	7	12	21	27	19	11	3

Soln

x	f	fx
1	7	7
2	12	24
3	21	63
4	27	108
5	19	95
6	11	66
7	3	21
	$\Sigma f = 100$	$\Sigma fx = 384$

$$A.M\ \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Or} \quad \frac{384}{100} = \frac{384}{100}$$

$$\bar{x} = 3.84$$

Median

x	f	c.f.
1	7	7
2	12	19
3	21	40
4	27	67
5	19	86
6	11	97
7	3	100

median $= \frac{n}{2} = \frac{100}{2} = 50^{th}$ value

$\boxed{x = 6}$

maximum frequency 27 occurs against $x = 4$

$\boxed{\text{mode} = 4}$

ex = Calculate the mode the following data ↓

class interval	frequency
0 - 10	10
10 - 20	14
20 - 30	19
30 - 40	17
40 - 50	13

Maximum frequency = 19

$f_m = 19$ $L = 20$ $f_{-1} = 14$

$f_1 = 17$ $i = 10$

$$\text{Mode} = L + \frac{f_m - f_{-1}}{2f_m - f_{-1} - f_1} \times i$$

$$= 20 + \frac{19 - 14}{2 \times 19 - 14 - 17} \times 10$$

$$= 20 + \frac{5}{7} \times 10$$

$$= 20 + 0.714 \times 10$$

$$= 20 + 7.142$$

$$\boxed{= 27.14}$$

Ex The following data is the Result of 400 HIV positive men in a Reported age group of India. Calculate the mode from the given data

Age limit	frequency
20-24	2
25-29	85
30-34	135
35-39	91
40-44	57
45-49	20
50-54	10

Maximum frequency $f_m = 135$

$L = 30 \quad f_{-1} = 85 \quad f_1 = 91$

$i = 4$

$$\text{mode} = L + \frac{f_m - f_{-1}}{2f_m - f_{-1} - f_1} \times i$$

$$= 30 + \frac{135 - 85}{2 \times 135 - 85 - 91} \times 4$$

$$= 30 + \frac{50}{94} \times 4$$

$$= 30 + 0.5319 \times 4$$

$$= 30 + 2.1276$$

$$= 32.127$$

ex Fine the mode for the followings frequency distribution

size	Demand
Below 10	3
10 - 12	15
12 - 14	27
14 - 16	20
16 - 18	3
18 - 20	2

Maximum frequency $fm = 27$

$L = 12 \quad f_{-1} = 15 \quad f_1 = 20 \quad i = 2$

$$\text{Mode} = L + \frac{fm - f_{-1}}{2 \times fm - f_1 - f_1} \times i$$

$$= 12 + \frac{27 - 15}{2 \times 27 - 15 - 20} \times 2$$

$$= 12 + \frac{12}{a.11} \times 2$$

$$= 12 + 0.6315 \times 2$$

$$= 12 + 1.263$$

$$\boxed{= 13.26}$$